ALLOWANCE FOR BULK RELAXATION BY THE METHOD OF INTERNAL FRICTION

S. I. Meshkov and G. N. Pachevskaya

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There is presently no reliable experimental evidence for the relaxation spectra of solids due to purely bulk deformations, because direct measurement of basic relaxation characteristics (frequency and modulus defect) involves major difficulties. For this reason, even the existence of bulk relaxation is not entirely clear, in spite of many statements [1–3] that there is a second, or bulk, viscosity.

The internal-friction method is valuable in the study of relaxation processes, and for shear deformation (torsion pendulum) it allows one to compare each relaxation process with a rheological model and a physical mechanism [4]. It is difficult to isolate bulk relaxation in pure form, so it is of interest to take into account its effects on shear relaxation with reference to the longitudinal oscillations of a specimen.

The relaxation spectrum can sometimes be described in the theory of elasticity by means of Rabotnov's fractional exponentials [5]. Then the stress tensor σ_{ik} of a homogeneous isotropic solid (neglecting thermal relaxation) is

$$\sigma_{ik} = K_\infty \delta_{ik} \varepsilon_{ll} - \Delta K \delta_{ik} \int_{-\infty}^{t} \varepsilon_{ll}(t')\, \vartheta_{\gamma_2}(t-t', \tau_{\varepsilon 2})\, dt' + 2\mu_\infty e_{ik} - 2\Delta\mu \int_{-\infty}^{t} e_{ik}(t')\, \vartheta_{\gamma_1}(t-t', \tau_{\varepsilon 1})\, dt', \qquad (1)$$

in which e_{ik} is the deviator of the deformation tensor, δ_{ik} is the Kronecker symbol, $\Delta K = K_\infty - K_0$ and $\Delta\mu = \mu_\infty - \mu_0$ are the differences between the unrelaxed and relaxed values of the bulk and shear moduli, respectively, and ϑ_γ is Rabotnov's fractional-exponential relaxation kernel:

$$\vartheta_\gamma(t,\tau) = \frac{1}{\tau_\varepsilon}\left(\frac{t}{\tau_\varepsilon}\right)^{\gamma-1} \sum_{n=0}^{\infty} \frac{(-1)^n (t/\tau_\varepsilon)^{\gamma n}}{\Gamma[\gamma(n+1)]} \qquad (0<\gamma\leqslant 1), \quad (2)$$

where the subscript 1 relates to shear relaxation and the subscript 2 to bulk relaxation.

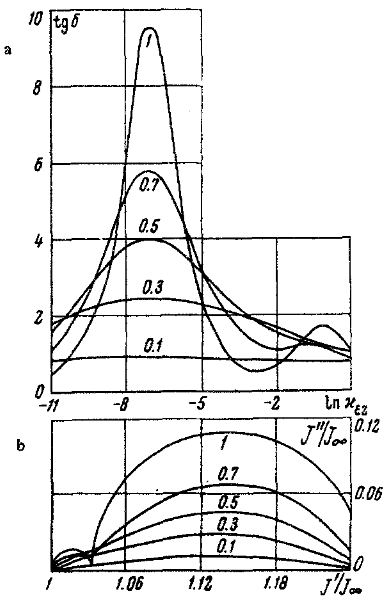

Fig. 1

We can apply (1) to torsional oscillations of small amplitude, when the hypothesis of planar sections applies, to explain the broad relaxation peak for internal friction [6]. The effects of bulk relaxation can be seen in the steady-state longitudinal oscillations of a specimen whose material is described by (1), for which purpose it is convenient to rewrite (1) in the space of Laplace transforms [9]:

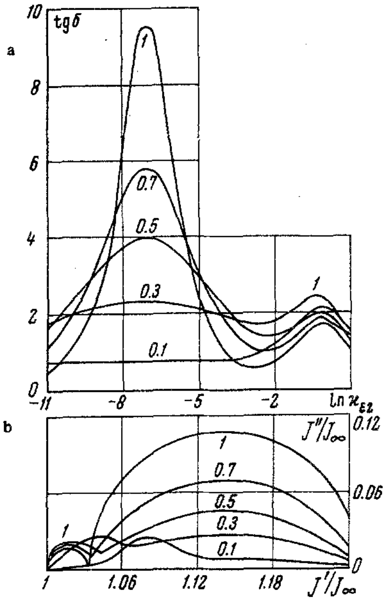

Fig. 2

$$\sigma_{ik}^{**} = K^* \varepsilon^{**} \delta_{ik} + 2\mu^* e_{ik}^{**}, \qquad (3)$$

in which a single asterisk denotes the transformation of a one-sided Laplace transformation, while two asterisks do the same for a two-sided one. The initial relation between the stress and deformation tensors then corresponds to an elastic problem, only with the difference that the elastic moduli are dependent on the parameter p:

$$K^* = K_\infty - \Delta K\, [1 + (p\tau_{\varepsilon 2})^{\gamma_2}]^{-1}, \qquad (4)$$

$$\mu^* = \mu_\infty - \Delta\mu\, [1 + (p\tau_{\varepsilon 1})^{\gamma_1}]^{-1}. \qquad (5)$$

The subsequent discussion of longitudinal extension of a rod corresponds exactly to the elastic problem [7], which gives us the following for the compliance and Poisson's ratio:

$$J^*(p) = \frac{1}{3}\left(\frac{1}{\mu^*} + \frac{1}{3K^*}\right), \qquad \nu^*(p) = \frac{1}{2}\,\frac{3K^* - 2\mu^*}{3K^* + \mu^*}. \qquad (6)$$

In (6) we convert to the space of Fourier transforms $p \to i\omega$, to get the following complex expressions for the compliance and Poisson's ratio in stationary periodic tests:

$$J(\omega) = J'(\omega) - iJ''(\omega), \qquad \nu(\omega) = \nu'(\omega) - i\nu''(\omega), \quad (7)$$

in which

$$J' = {}^1/_3\, J_\infty [2(1+\nu_\infty) L_1 + (1-2\nu_\infty) L_2],$$

$$J'' = {}^1/_3\, J_\infty [2(1+\nu_\infty) M_1 + (1-2\nu_\infty) M_2, \qquad (8)$$

$$L_j = \frac{\varkappa_{\sigma j}^{\gamma_j} + \varkappa_{\varepsilon j}^{-\gamma_j} + [1 + (\varkappa_{\sigma j}/\varkappa_{\varepsilon j})^{\gamma_j}] \cos\psi_j}{\varkappa_{\sigma j}^{\gamma_j} + \varkappa_{\sigma j}^{-\gamma_j} + 2\cos\psi_j},$$

$$M_j = \frac{[(\varkappa_{\sigma j}/\varkappa_{\varepsilon j})^{\gamma_j} - 1]\sin\psi_j}{\varkappa_{\sigma j}^{\gamma_j} + \varkappa_{\sigma j}^{-\gamma} + 2\cos\psi_j},$$

$$\varkappa_{\sigma j} = \omega\tau_{\sigma j}, \quad \varkappa_{\varepsilon j} = \omega\tau_{\varepsilon j}, \quad \psi_j = {}^1/_2\gamma_j\pi \quad (j = 1, 2), \quad (9)$$

where J_∞ and ν_∞ are the unrelaxed values of the compliance and Poisson's ratio.

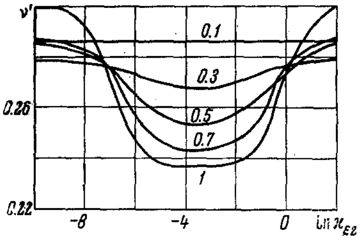

Fig. 3

We have the following for the real part of Poisson's ratio:

$$\nu' = 0.5\,(a_2b_1 - \xi_1 c - 2\xi_1\xi_2\, a_1b_2) \times$$
$$\times\,(a_2b_1 + 2\xi_1 c + \xi_1\xi_2\, a_1b_2)^{-1}, \quad (10)$$

$$a_j = \varkappa_{\sigma j}^{\gamma_j} + \varkappa_{\sigma j}^{-\gamma_j} + 2\cos\psi_j, \quad b_j = \varkappa_{\varepsilon j}^{\gamma_j} + \varkappa_{\varepsilon j}^{-\gamma_j} + 2\cos\psi_j,$$

$$c = \{\varkappa_{\varepsilon 2}^{\gamma_2} + [1 + (\varkappa_{\varepsilon 2}/\varkappa_{\sigma 2})^{\gamma_2}]\cos\psi_2 + \varkappa_{\sigma 2}^{-\gamma_2}\}\{\varkappa_{\varepsilon 1}^{\gamma_1} +$$
$$+ [1 + (\varkappa_{\varepsilon 1}/\varkappa_{\sigma 1})^{\gamma_1}]\cos\psi_1 + \varkappa_{\sigma 1}^{-\gamma_1}\} +$$
$$+ [1 - (\varkappa_{\varepsilon 2}/\varkappa_{\sigma 2})^{\gamma_2}]\,[1 - (\varkappa_{\varepsilon 1}/\varkappa_{\sigma 1})^{\gamma_1}]\sin\psi_1\sin\psi_2, \quad (11)$$

in which we have used the ratios

$$\mu_0/\mu_\infty = (\tau_{\varepsilon 1}/\tau_{\sigma 1})^{\gamma_1},$$

$$K_0/K_\infty = (\tau_{\varepsilon 2}/\tau_{\sigma 2})^{\gamma_2}, \quad \xi_1 = \mu_\infty/3K_0, \quad \xi_2 = \mu_0/3K_\infty.$$

We take as measure of the internal friction the tangent of the phase shift between the stress and deformation as given by (8):

$$\operatorname{tg}\delta = J''/J'. \quad (12)$$

We consider a numerical example with $\nu_\infty = 0.3$, $\mu_0/\mu_\infty = K_0/K_\infty = 0.8$, $\tau_{\varepsilon 1}/\tau_{\varepsilon 2} = 10^3$, $\omega = 1$ in order to establish whether, in principle, a relaxation peak due to bulk deformation can occur. Figure 1 shows as $\ln \varkappa_{\varepsilon 2} \sim T^{-1}$ the tan δ and phase diagram for the compliance $J'' = f(J')$ (parts a and b, respectively). As parameter we have $\gamma = \gamma_1 = \gamma_2$, the values being given on the curves. The limiting value $\gamma = 1$ indicates that the shear and bulk relaxations are described by models of a standard linear body, and it leads to clear separation of the two peaks. If the bulk and shear moduli have equal degrees of relaxation, they make unequal contributions to the total effect, as defined by ν_∞, the shear peak being about 5.5 times larger than the bulk one. Reduction in γ corresponds to broadening of the relaxation spectrum [6], and the difference between the peaks vanishes. The value $\gamma = 0.5$, which corresponds to a relaxation kernel expressed by means of the probability integral, may be considered in this case to correspond to the lower limit for the bulk effect, i.e., bulk relaxation is not seen for all $\gamma < 0.5$, although it exists. The possibility of observing bulk relaxation is thus dependent not only on the ratio of the relaxation times for shear and bulk stresses [8], but also on the parameter that characterizes the width of the relaxation spectrum.

The curves of Fig. 2 are analogous to those of Fig. 1, but illustrate the situation where $\gamma_1 \neq \gamma_2$. Here we have assumed $\gamma_2 = 1$, i.e., the bulk relaxation obeys the model of a standard linear body, while the values of γ_1 are given on the curves. The broadening may be so great as to lead to loss of the shear peak, e.g., for $\gamma_1 = 0.1$.

Finally, Fig. 3 shows that bulk relaxation clearly affects the behavior of the static Poisson's ratio ν'. The curves for $\gamma = \gamma_1 = \gamma_2$ show that bulk relaxation reduces ν', while shear relaxation restores it to its unrelaxed value ν_∞. The effect increases with γ and is largest for $\gamma = 1$.

In the limit $\tau_{\varepsilon 2} \to \infty$, when there is only shear relaxation, the latter increases Poisson's ratio from ν_∞ to a value dependent on the degree of relaxation of the shear modulus.

As $\ln \varkappa_{\varepsilon 2} \sim T^{-1}$, the temperature dependence of the dynamic Poisson's ratio is the most convenient means of detecting bulk relaxation.

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